

**Directions:** Select the best response to each problem below. Items marked NC are not permitted a calculator to answer the problem and those marked C may require the use of a calculator to solve the problem.

|                        |   |
|------------------------|---|
| 1. _____ NC<br>'08 #1  | <p>At time <math>t \geq 0</math>, a particle moving in the <math>xy</math>-plane has velocity vector given by <math>v(t) = \langle t^2, 5t \rangle</math>. What is the acceleration vector of the particle at time <math>t = 3</math>?</p> <p>(A) <math>\langle 9, \frac{45}{2} \rangle</math>      (B) <math>\langle 6, 5 \rangle</math>      (C) <math>\langle 2, 0 \rangle</math>      (D) <math>\sqrt{306}</math>      (E) <math>\sqrt{61}</math></p>   |
| 2. _____ NC<br>'08 #6  | $f(x) = \begin{cases} \frac{x^2 - 4}{x - 2} & \text{if } x \neq 2 \\ 1 & \text{if } x = 2 \end{cases}$ <p>Let <math>f</math> be the function defined above. Which of the following statements about <math>f</math> are true?</p> <p>I. <math>f</math> has a limit at <math>x = 2</math>.</p> <p>II. <math>f</math> is continuous at <math>x = 2</math>.</p> <p>III. <math>f</math> is differentiable at <math>x = 2</math>.</p> <p>(A) I only<br/>(B) II only<br/>(C) III only<br/>(D) I and II only<br/>(E) I, II, and III</p> |
| 3. _____ NC<br>'08 #15 | <p>If <math>f(x) = (\ln x)^2</math>, then <math>f''(\sqrt{e}) =</math></p> <p>(A) <math>\frac{1}{e}</math>      (B) <math>\frac{2}{e}</math>      (C) <math>\frac{1}{2\sqrt{e}}</math>      (D) <math>\frac{1}{\sqrt{e}}</math>      (E) <math>\frac{2}{\sqrt{e}}</math></p>  |
| 4. _____ NC<br>'08 #17 | <p>Let <math>h</math> be a differentiable function, and let <math>f</math> be the function defined by <math>f(x) = h(x^2 - 3)</math>. Which of the following is equal to <math>f'(2)</math>?</p> <p>(A) <math>h'(1)</math>      (B) <math>4h'(1)</math>      (C) <math>4h'(2)</math>      (D) <math>h'(4)</math>      (E) <math>4h'(4)</math></p>   |
| 5. _____ NC<br>'08 #18 | <p>In the <math>xy</math>-plane, the line <math>x + y = k</math>, where <math>k</math> is a constant, is tangent to the graph of <math>y = x^2 + 3x + 1</math>. What is the value of <math>k</math>?</p> <p>(A) <math>-3</math>      (B) <math>-2</math>      (C) <math>-1</math>      (D) <math>0</math>      (E) <math>1</math></p>   |
| 6. _____ NC<br>'08 #25 | $f(x) = \begin{cases} cx + d & \text{for } x \leq 2 \\ x^2 - cx & \text{for } x > 2 \end{cases}$ <p>Let <math>f</math> be the function defined above, where <math>c</math> and <math>d</math> are constants. If <math>f</math> is differentiable at <math>x = 2</math>, what is the value of <math>c + d</math>?</p> <p>(A) <math>-4</math>      (B) <math>-2</math>      (C) <math>0</math>      (D) <math>2</math>      (E) <math>4</math></p>  |
| 7. _____ NC<br>'08 #28 | <p>In the <math>xy</math>-plane, a particle moves along the parabola <math>y = x^2 - x</math> with a constant speed of <math>2\sqrt{10}</math> units per second. If <math>\frac{dx}{dt} &gt; 0</math>, what is the value of <math>\frac{dy}{dt}</math> when the particle is at the point <math>(2, 2)</math>?</p> <p>(A) <math>\frac{2}{3}</math>      (B) <math>\frac{2\sqrt{10}}{3}</math>      (C) <math>3</math>      (D) <math>6</math>      (E) <math>6\sqrt{10}</math></p>   |

8. \_\_\_\_\_ C  
'08 #85

A particle moves on the  $x$ -axis with velocity given by  $v(t) = 3t^4 - 11t^2 + 9t - 2$  for  $-3 \leq t \leq 3$ . How many times does the particle change direction as  $t$  increases from  $-3$  to  $3$ ?

(A) Zero      (B) One      (C) Two      (D) Three      (E) Four

9. \_\_\_\_\_ C  
'08#86

On the graph of  $y = f(x)$ , the slope at any point  $(x, y)$  is twice the value of  $x$ . If  $f(2) = 3$ , what is the value of  $f(3)$ ?

(A) 6      (B) 7      (C) 8      (D) 9      (E) 10

10. \_\_\_\_\_ C  
'08 #90

| $x$ | $f(x)$ | $g(x)$ | $f'(x)$ | $g'(x)$ |
|-----|--------|--------|---------|---------|
| -1  | -5     | 1      | 3       | 0       |
| 0   | -2     | 0      | 1       | 1       |
| 1   | 0      | -3     | 0       | 0.5     |
| 2   | 5      | -1     | 5       | 2       |

The table above gives values of the differentiable functions  $f$  and  $g$  and of their derivatives  $f'$  and  $g'$ , at selected values of  $x$ . If  $h(x) = f(g(x))$ , what is the slope of the graph of  $h$  at  $x = 2$ ?

(A) -10      (B) -6      (C) 5      (D) 6      (E) 10

11. \_\_\_\_\_ C  
'08 #92

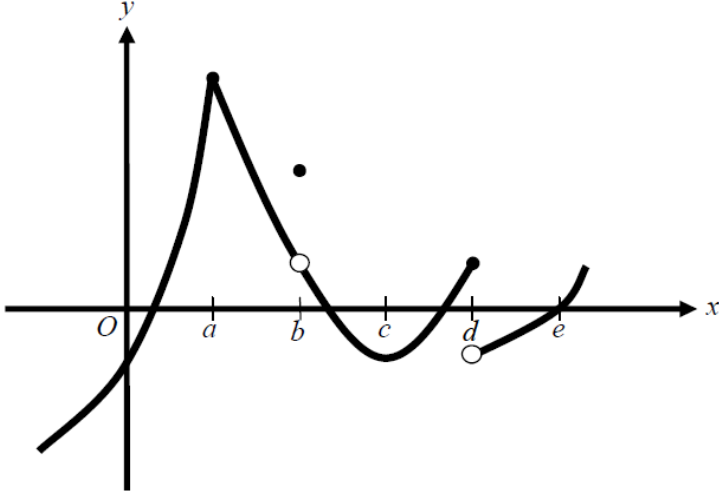
The figure above shows the graphs of the functions  $f$  and  $g$ . The graphs of the lines tangent to the graph of  $g$  at  $x = -3$  and  $x = 1$  are also shown. If  $B(x) = g(f(x))$ , what is  $B'(-3)$ ?

(A)  $-\frac{1}{2}$       (B)  $-\frac{1}{6}$       (C)  $\frac{1}{6}$       (D)  $\frac{1}{3}$       (E)  $\frac{1}{2}$

12. \_\_\_\_\_ NC  
'03 #1

If  $y = \sin(3x)$  then  $\frac{dy}{dx} =$

(A)  $-3\cos(3x)$   
 (B)  $-\cos(3x)$   
 (C)  $-\frac{1}{3}\cos(3x)$   
 (D)  $\cos(3x)$   
 (E)  $3\cos(3x)$

|                     |  |
|---------------------|--|
| 13. ____ NC '03 #4  | <p>For <math>0 \leq t \leq 13</math> an object travels along an elliptical path given by the parametric equations <math>x = 3\cos t</math> and <math>y = 4\sin t</math>. At the point where <math>t = 13</math>, the object leaves the path and travels along the line tangent to the path at that point. What is the slope of the line on which the object travels?</p> <p>(A) <math>-\frac{4}{3}</math><br/>           (B) <math>-\frac{3}{4}</math><br/>           (C) <math>-\frac{4\tan 13}{3}</math><br/>           (D) <math>-\frac{4}{3\tan 13}</math><br/>           (E) <math>-\frac{3}{4\tan 13}</math></p> |
| 14. ____ NC '03 #9  | <p>If <math>f(x) = \ln(x + 4 + e^{-3x})</math>, then <math>f'(0)</math> is</p> <p>(A) <math>-\frac{2}{5}</math><br/>           (B) <math>\frac{1}{5}</math><br/>           (C) <math>\frac{1}{4}</math><br/>           (D) <math>\frac{2}{5}</math><br/>           (E) <i>nonexistent</i></p>  |
| 15. ____ NC '03 #13 |  <p>The graph of a function <math>f</math> is shown above. At which value of <math>x</math> is <math>f</math> continuous, but not differentiable?</p> <p>(A) <b>a</b>      (B) <b>b</b>      (C) <b>c</b>      (D) <b>d</b>      (E) <b>e</b></p>   |
| 16. ____ NC '03 #16 | <p>If the line tangent to the graph of the function <math>f</math> at the point <math>(1, 7)</math> passes through the point <math>(-2, -2)</math>, then <math>f'(1)</math> is</p> <p>(A) <math>-5</math><br/>           (B) <math>1</math><br/>           (C) <math>3</math><br/>           (D) <math>7</math><br/>           (E) <i>undefined</i></p>  |

17. \_\_\_\_ NC  
'03 #17
- A curve  $C$  is defined by the parametric equations  $x = t^2 - 4t + 1$  and  $y = t^3$ . Which of the following is an equation of the line tangent to the graph of  $C$  at the point  $(-3, 8)$ ?
- (A)  $x = -3$   
 (B)  $x = 2$   
 (C)  $y = 8$   
 (D)  $y = -\frac{27}{10}(x+3) + 8$   
 (E)  $y = 12(x+3) + 8$

18. \_\_\_\_ C  
'03 #79

| $x$ | $f(x)$ | $f'(x)$ | $g(x)$ | $g'(x)$ |
|-----|--------|---------|--------|---------|
| -1  | 6      | 5       | 3      | -2      |
| 1   | 3      | -3      | -1     | 2       |
| 3   | 1      | -2      | 2      | 3       |

The table above gives values of  $f$ ,  $f'$ ,  $g$ , and  $g'$  at selected values of  $x$ . If  $h(x) = f(g(x))$ , then  $h'(1) =$

- (A) 5  
 (B) 6  
 (C) 9  
 (D) 10  
 (E) 12

19. \_\_\_\_ C  
'03 #84

A particle moves in the  $xy$ -plane so that its position at any time  $t$  is given by  $x(t) = t^2$  and  $y(t) = \sin(4t)$ . What is the speed of the particle when  $t = 3$ ?

- (A) 2.909  
 (B) 3.062  
 (C) 6.884  
 (D) 9.016  
 (E) 47.393

20. \_\_\_\_ C  
'03 #91

The height  $h$ , in meters, of an object at time  $t$  is given by  $h(t) = 24t + 24t^{3/2} - 16t^2$ . What is the height of the object at the instant when it reaches its maximum upward velocity?

- (A) 2.545 meters  
 (B) 10.263 meters  
 (C) 34.125 meters  
 (D) 54.889 meters  
 (E) 89.005 meters